



DYNAMIC AND SECURITY-CONSTRAINED ASSESSMENT OF PHOTOVOLTAIC HOSTING CAPACITY IN A LOW-INERTIA POWER SYSTEM: IEEE 39-BUS VALIDATION AND EMPIRICAL APPLICATION TO SENEGAL

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ABSTRACT

As photovoltaic (PV) penetration rises across interconnected grids, hosting capacity (HC) can no longer be inferred solely from voltage margins and equipment-loading constraints. The combined effects of reduced synchronous inertia and solar variability require a unified treatment of steady-state operating conditions, contingencies, uncertainty, and short-term frequency dynamics. This study proposes a hierarchical assessment methodology that integrates AC power-flow limits, N–1 security, PV and load uncertainty, and frequency-security criteria. The workflow is first demonstrated on the IEEE New England 39-bus system and subsequently aligned with Senegalese operating records and historical PATRP analyses. On the test system, spatially dispersed PV injections yield a static HC of 1,400 MW, whereas concentrating PV at fewer buses reduces the value to approximately 1,175 MW, a decrease near 19%. Under the selected non-radial contingencies, HCN–1 remains at 1,400 MW when benchmark emergency limits are applied. With 500 Monte Carlo realizations per operating point and relative standard deviations of 5% for PV and 1% for load, the probabilistic HC, HC_{prob}, is about 975, 1,100, and 1,150 MW for risk tolerances of 1%, 5%, and 10%. Frequency performance depends strongly on external support: after displacing 2,000 MW of synchronous generation, the simulated nadir improves from roughly 49.529 Hz without external support to 49.706 Hz with an effective 25% support level. Senegalese observations for 12–14 August 2025 indicate generally stable operation during the period assessed, including a measured PV peak of 188 MW and over 95% of frequency samples within 49.7–50.3 Hz. While these observations clarify the operating context, they do not suffice to produce a national HC estimate in the absence of a complete, validated SPS-2025 model. The principal contribution is therefore a transparent and reproducible protocol that disaggregates static, probabilistic, contingency-related, and frequency-related limits prior to its full-scale application to the Senegalese transmission network.

KEYWORDS: photovoltaic hosting capacity; low-inertia systems; frequency security; N–1 contingency; uncertainty quantification; Senegal; West Africa; power system stability.

Nomenclature and Abbreviations

HC: hosting capacity; PV: photovoltaic; BESS: battery energy storage system; N–1: loss of a single system element; OMVS: Organisation pour la Mise en Valeur du fleuve Sénégal; OMVG: Organisation pour la Mise en Valeur du fleuve Gambie; PATRP: Power Africa Transactions and Reforms Program; SPS: power-system model used for stability analyses; RoCoF: rate of change of frequency; E_{kin}: synchronous kinetic energy; H_{eq}: equivalent inertia constant.

1. INTRODUCTION

The accelerating deployment of converter-based renewable resources is reshaping both long-term planning and real-time operation of power systems. While large-scale PV can reduce fuel consumption and associated emissions, it also reconfigures active and reactive power flows and may reduce the number of synchronized machines online. Once synchronous plant commitment declines, voltage margins, contingency response, and frequency dynamics cease to be separable

concerns and must be assessed jointly.

Hosting capacity is typically defined as the maximum PV power that can be integrated without violating specified operating criteria. The literature examines this boundary through deterministic, stochastic, and chronological formulations.^[5, 8] Earlier work emphasized distribution-level constraints, including overvoltage, thermal limits, and reverse power flow. More recent analyses incorporate temporal variability and system dynamics. For example, a 2026 study of Niger demonstrates the value of full chronological AC simulations for an emerging West African system^[2], and other studies show that frequency-security requirements can materially reduce an HC that would appear acceptable under network-only criteria.^[3] In weak or low-inertia grids, the relevant construct is therefore better framed as a multi-criteria security envelope rather than a single voltage or thermal threshold.

Senegal offers a pertinent case study. In 2025, installed capacity totaled 2,310.5 MW and peak demand reached 1,261 MW.^[11] At 13:00 on 13 August 2025, solar output peaked at 188 MW, and frequency remained within 49.7–50.3 Hz for more than 95% of the period examined.^[11] The same year marked the commissioning of Walo Storage, the first utility-scale BESS in the country. Senegal also participates in regional exchanges through the OMVS and OMVG interconnections. In this context, the attainable PV hosting level depends on interconnection status, available reserves, and synchronous-unit commitment.

Historical stability assessments lead to the same conclusion. The PATRP program highlighted the interactions between renewable variability, spinning reserves, regional inerties, and frequency response.^[13] Its 2017–2035 study presented N–1 analysis of the principal 225- and 90-kV corridors, examining voltage, transient, and frequency stability under severe events. A key inference was that regional interconnections can furnish inertial support, yet do not eliminate the need for adequate local spinning reserves.^[13]

Within this setting, the guiding question is direct: how does the estimated upper bound on PV hosting capacity shift when static constraints, N–1 security, solar and load uncertainty, and declining synchronous inertia are evaluated in an integrated manner for an emerging interconnected system?

1.1. Contributions

- A four-tier framework for PV hosting-capacity assessment that unifies static AC limits, N–1 security screening, probabilistic uncertainty, and frequency-security verification.
- A rigorous separation between PV penetration with respect to demand and PV share within the generation portfolio, avoiding ambiguity in reporting HC.

- A low-inertia formulation that maps PV-driven displacement of synchronous generation to equivalent kinetic energy, RoCoF, and frequency nadir outcomes.
- A two-step validation sequence comprising application to the IEEE 39-bus system followed by calibration and testing on the Senegalese transmission grid using official operating data.
- A Senegal-focused validation case reproducing the observed August 2025 renewable episode, including a 188 MW solar peak and a subsequent PV decline induced by cloud cover.

2. Literature Review and Research Gap

Recent work on hosting capacity converges on a common conclusion: relying on a single deterministic operating point is inadequate when solar production is variable. Chronological AC simulations on the Niger River area grid demonstrated that hosting capacity is a joint outcome of demand levels, PV availability, and voltage-control actions.^[2] Probabilistic approaches broaden this view by quantifying the probability of constraint violations under simultaneous uncertainty in PV injections and load. Monte Carlo analyses on operational networks similarly reveal that siting, plant scale, and load variability can materially displace the hosting-capacity frontier.^[1, 6, 7, 14, 17]

Although storage and advanced control strategies can alleviate several of these constraints^[18], that is not the initial focus of this paper. We deliberately distinguish between diagnosis and mitigation. Article 1 delineates the intrinsic and secure PV hosting boundary and identifies the mechanism that renders it binding; subsequent papers then assess how BESS sizing, siting, and operation modify that boundary. Keeping these phases separate allows any observed improvement to be attributed to the flexibility resource rather than to changes in the underlying network representation.

Frequency security has also taken on a more prominent role in hosting-capacity analysis. Recent studies explicitly embed post-disturbance constraints or grid-forming support within the hosting-capacity formulation.^[3, 9] Nevertheless, much of the available evidence still originates from distribution systems, standard test cases, or regions outside sub-Saharan Africa. For interconnected West African systems, the combined impacts of PV uncertainty, displacement of synchronous units, regional support, spinning reserve, and N–1 security remain comparatively under-documented. These interactions are precisely those that become decisive when a nominal PV limit is conditioned by frequency performance.

The gap addressed here is therefore not a shortage of hosting-capacity methodologies, nor a lack of studies on low-inertia systems taken in isolation. What is missing is a unified assessment framework that evaluates static constraints, contingencies, uncertainty, and frequency

behavior under a coherent set of assumptions tailored to an emerging West African power system.

3. System Context and Data

3.1. Senegalese Power-System Context

In 2025, total electricity production in Senegal was estimated at 7,964 GWh for a national demand of 7,254 GWh, with an annual peak of 1,261 MW recorded on 4 November at 23:30.^[11] Installed capacity was approximately 2,310.5 MW, notably following the commissioning of West African Energy and Walo Storage.^[11] The 2025 Energy Information System report, which documents the 2024 energy balance, attributes 9.06% of national generation to solar power.^[12] We adopt this figure as a 2024 generation-mix reference point, not as a 2025 operating value.

The 12–14 August 2025 operating window offers a concrete test for confronting the proposed methodology with observed behavior. At 13:00 on 13 August, aggregate output from solar plants reached 188 MW. On the following day, widespread cloud cover drove a marked reduction in PV output. During the same period, Walo Storage operated in the evening, while PETN supported voltage regulation by absorbing reactive power.^[11] This sequence does not, on its own, determine a hosting-capacity value; it does, however, substantiate that the modeled mechanisms align with conditions actually encountered in operation.

The Senegalese grid must also be analyzed within its regional setting. The OMVS and OMVG interconnections are therefore included either explicitly or via validated dynamic equivalents, depending on the

level of detail required for the simulation. This modeling choice is essential, as historical analyses indicate that neighboring systems can contribute materially to system inertia and dynamic response.^[13]

3.2. Data Provenance

All model parameters are assigned a provenance class: A denotes measured or operator data; B denotes official documentary sources; C denotes literature or manufacturer information; D denotes research assumptions or sensitivity parameters. No class C or D value is presented as a measured Senelec parameter.

3.3. Historical Reference Parameters for Dynamic Calibration

The PATRP/Power Africa report provides quantitative benchmarks that are useful for checking the internal consistency of the Senegalese dynamic model.^[13] These values are not intended to describe the 2025 system; they are class-B historical data used for validation of the SPS-PATRP model and for sensitivity analyses. The study considered the 2019, 2022, and 2028 horizons, explicitly accounting for OMVS and OMVG contributions, spinning reserves, and severe generation-loss events.

For neighboring systems, the report assumes synchronous reserve equal to 3% of installed capacity. The corresponding historical values are 8 MW for OMVS in 2019, 12 MW for OMVS and 26 MW for OMVG in 2022, and 21 MW for OMVS and 44 MW for OMVG in 2028. Under this representation, total regional reserve increases from 8 MW in 2019 to 38 MW in 2022 and 65 MW in 2028.

Table 1: Historical synchronous-reserve benchmarks for neighboring systems used to validate the SPS-PATRP model (source: PATRP/Power Africa^[13]).

Horizon	OMVS installed (MW)	OMVS reserve (MW)	OMVG installed (MW)	OMVG reserve (MW)	Total regional reserve (MW)
2019	260	8	–	–	8
2022	400	12	883	26	38
2028	694	21	1 474	44	65

The report classifies as dimensioning contingencies the outage of the Sendou unit (115 MW), the worst-case loss of the Taïba wind farm (up to 72 MW in the 2022 studies), and the disconnection of a solar facility in the 47–60 MW range. It additionally quantifies potential maximum variations of 41 MW for the largest solar plant and 54 MW for the largest wind installation. These figures are maintained as historical benchmarks to inform contingency screening and to validate frequency-response performance, without being regarded as applicable regulatory thresholds. To complete this historical contingency portfolio, the preparation workbook also includes the loss of C404–C405 at Cap des Biches (31.5 MW) and the loss of TAG-4 at Bélair (30 MW). The Mékhé (60 MW) and Niakhar (47 MW) solar plants, along with the 2019 Taïba maximum of 47.6 MW, are preserved as reference points for reconstructing

the historical case. In line with the broader PATRP dataset, these values are employed for calibration and case selection and are not interpreted as parameters of the 2025 network.

4. METHODOLOGY

4.1. Definition of PV Penetration

The primary HC indicator is the penetration ratio relative to load.

$$\beta_{PV}(t) = PPV(t) / P_{Charge}(t)$$

A secondary indicator relative to the generation mix is also reported.

$$\alpha_{PV}(t) = PPV(t) / [PPV(t) + P_{synchron}(t) + P_{autre}(t)]$$

Both indicators are reported because the same installed PV capacity can correspond to different penetration

levels depending on demand and synchronous-unit commitment.

Table 2: Calculation criteria and parameters used in the study protocol.

Parameter	Symbol	Value	Unit	Class	Status / use
Nominal frequency	f_0	50	Hz	B	Network reference
Normal minimum voltage	V_{min}	0,95	pu	D	Methodological criterion to be confirmed against the grid code
Normal maximum voltage	V_{max}	1,05	pu	D	Methodological criterion to be confirmed against the grid code
Normal maximum loading	LoadingMax	100	%	D	Conventional limit
Frequency band - minimum	$f_{min,op}$	49,7	Hz	B	2025 operating reference
Frequency band - maximum	$f_{max,op}$	50,3	Hz	B	2025 operating reference
Probabilistic tolerance	ϵ_{risk}	1 / 5 / 10	%	D	Sensitivity cases; non-regulatory

4.2. Static AC Hosting Capacity

For a PV scaling variable λ , conventional static hosting capacity is defined as.

$$CA_{statique} = \max\{PPV(\lambda) : g_{AC}(x, \lambda) = 0, 0,95 \leq V_i \leq 1,05 \text{ pu}, \text{Chargement}_i \leq 100 \%, Q_{min} \leq Q_g \leq Q_{max}\}$$

The PV level is first swept using a coarse step and then refined around the first violation. Recorded outputs include V_{min} , V_{max} , the number of voltage violations, maximum line/transformer loading, network losses, and reactive-power limit violations.

4.3. N-1 Secure Hosting Capacity

For each PV level, selected single contingencies are applied to the transmission network. The secure value is: $HCN-1 = \max\{PPV : \text{all selected single contingencies satisfy emergency operating criteria}\}$

The security penalty is quantified as $\Delta HCN-1 = (HC_{static} - HCN-1)/HC_{static} \times 100\%$. Candidate contingencies include the loss of a large synchronous unit, a critical transmission circuit, an interconnection, and a major renewable injection.

4.4. Probabilistic Hosting Capacity

PV and load forecast errors are represented by scenario-dependent random variables: $PPV_{k(t)} = \hat{PPV}(t) + \epsilon_{PV,k(t)}$; $PL_{k(t)} = \hat{PL}(t) + \epsilon_{L,k(t)}$

Probabilistic HC is defined, for a selected risk tolerance ϵ_{risk} , as.

$$HC_{prob} = \max\{PPV : \Pr[\text{violation of an operating constraint}] \leq \epsilon_{risk}\}$$

Risk levels of 1%, 5%, and 10% are treated as sensitivity cases rather than regulatory thresholds. Monte Carlo convergence is checked by progressively increasing the number of scenarios.

4.5. Low-Inertia and Frequency-Security Model

The kinetic energy associated with online synchronous units is.^[4]

$$E_{cin} = \sum_i H_i S_i$$

and the equivalent inertia constant is.

$$H_{eq} = \sum_i H_i S_i / \sum_i S_i$$

For an active-power imbalance ΔP , the initial RoCoF can be approximated by.

$$RoCoF \approx -f_0 \Delta P / (2 E_{cin})$$

Subsequent dynamic studies are used to quantify the realized rate of change of frequency, the nadir, the maximum absolute deviation, and the time to frequency recovery. The operational frame of reference is the Senegalese band of 49.7–50.3 Hz, while event-driven under-frequency protection thresholds are treated as distinct from routine operating limits.

4.5.1. Regarding the dynamic representation of PV resources and the regional network

the historical SPS-PATRP formulation embeds dedicated dynamic models for generating units, aggregated loads, under-frequency load shedding, neighboring interconnections, and PV converters together with their controls. The detailed converter and controller parameters archived in the PATRP appendices are employed solely to reconstruct the historical case and to confirm end-to-end simulation consistency. In the SPS-2025 formulation, any more recent parameterization supersedes the historical data and is mapped to class A or class B as appropriate; in the absence of such updates, the corresponding quantities are relegated to class C or D for sensitivity analysis.

4.5.2. With respect to spinning reserve, contingencies, and the frequency criterion

spinning reserve is modeled explicitly so that the contribution of instantaneous inertia can be disentangled from primary frequency response. The historical analysis assumed 63 MW of thermal headroom distributed across units C6–C7 (24 MW), Malicounda (15 MW), Contour Global (10 MW), and Tobene (14 MW), with a potential supplementary regional contribution. For the 2022 horizon, the report indicated that the loss of 115 MW could necessitate approximately 70 MW of reserve, and

that about 54 MW of reserve might be required in certain scenarios to buffer renewable variability. In the present study these figures are retained strictly for validation and sensitivity exploration; they are not imposed as constraints on the 2025 system. Frequency-domain simulations therefore examine multiple combinations of reserve availability and imports. For each PV penetration level, the critical contingency is applied and the resulting RoCoF, nadir, recovery time, and any encroachment on under-frequency load-shedding thresholds are recorded. Interconnection effects are probed by varying, in isolation, the import level, the available primary reserve, and the regional inertial equivalent, thereby avoiding attribution of condition-dependent support to regional coupling per se.

4.6. Secure Hosting-Capacity Envelope

The secure hosting-capacity envelope is defined by

$$HC_{\text{secure}} = \min(HC_{\text{static}}, HCN-1, HC_{\text{prob}}, HC_{\text{frequency}}).$$

This construction isolates the first mechanism to become binding. To characterize the impact of frequency-security constraints on hosting capacity, the following indicator is defined.

$$FCHCR = (HC_{\text{static}} - HC_{\text{frequency}}) / HC_{\text{static}} \times 100\%.$$

FCHCR is reported together with the contingency, inertia, and reserve assumptions that prevail at the corresponding boundary.

4.7. Validation Architecture

The validation architecture proceeds in four stages. First, the New England 39-bus benchmark is used to establish reproducibility and to validate the algorithmic workflow, with HC_{static} , $HCN-1$, and the set of limiting buses and branches as primary outputs. Second, the historical Senegalese SPS-PATRP model is used to verify topological and dynamic consistency against 2017–2035 data, including flow patterns, contingency behavior, reserve representation, PV modeling, and historical frequency responses. Third, the SPS-2025 Senegal model is applied to the actual system, giving precedence to measured and official 2025 data, with HC_{secure} and the resulting hierarchy of constraints as main deliverables. Fourth, prospective PV configurations are explored through sensitivity analyses covering PV siting, synchronous commitment, PV and load uncertainty, reserve levels, and interconnection status.

4.8. Reproducibility and Traceability Protocol

The reproducibility and traceability protocol associates each simulation with a unique scenario identifier, a time stamp or load trajectory, a PV penetration level, a synchronous-unit commitment schedule, an interconnection state, and a parameter-provenance class. PV injections are increased until the first violation is detected, after which the search is refined around the feasibility boundary. In probabilistic assessments, the Monte Carlo sample size is increased until the estimated violation probability converges within tolerance. Values inherited from PATRP are preserved in a separate

historical dataset to prevent conflation with 2025 parameters. For the benchmark, static data are drawn from the New England IEEE 39-bus case distributed with MATPOWER (case39), and an AC power flow is solved using Newton–Raphson at each PV step. Criteria, contingencies, injection patterns, and probabilistic assumptions are specified in Section 7.1. Parameters lacking operator provenance are designated class D sensitivity assumptions. This segregation among reference data, assumptions, and Senegal-specific inputs ensures that benchmark-specific choices are not inadvertently ascribed to the actual network.

4.9. Calculation Algorithm and Stopping Rules

The calculation algorithm and associated stopping rules are organized hierarchically so that a hosting-capacity value obtained under a less stringent criterion is never interpreted as a secure limit. For each scenario, the base operating point is first validated. PV injections are then increased incrementally until the first AC-limit violation occurs; the operating boundary is subsequently refined to obtain HC_{static} . $N-1$ contingencies are applied to all PV levels that satisfy the base-case criteria to determine $HCN-1$. For probabilistic evaluations, PV and load forecast errors are sampled and violation probabilities are computed at prescribed risk tolerances. Admissible cases then undergo dynamic frequency simulations, with kinetic energy E_{kin} and equivalent inertia H_{eq} recalculated according to the synchronous commitment. $HC_{\text{frequency}}$ is defined as the largest PV level that satisfies the dynamic criteria. The final hosting capacity is the minimum across the four values. The stopping rule is coupled to the nature of the first binding constraint, whether a voltage limit, thermal overload, reactive capability bound, $N-1$ contingency, probabilistic risk threshold, or frequency-security criterion. The limiting bus, branch, device, or event is recorded together with the corresponding PV level to preserve a physically interpretable boundary.

5. Simulation Scenarios

The simulation suite comprises the following scenarios. S0 is the reference operating condition used for base power-flow validation. S1 represents a high-load, low-PV state characteristic of conventional peak-demand operation. S2 captures a low-load, high-PV condition expected to be critical due to potential PV export and reduced synchronous commitment. S3 combines high load with high PV to stress both the network and renewable integration. S4 represents a cloud-ramp event featuring rapid PV reduction and is used to probe sensitivity to frequency dynamics and reserve sufficiency. S5 consists of a representative time series for probabilistic and time-dependent hosting-capacity estimation. SEN-AUG2025 denotes the 12–14 August 2025 window selected for empirical validation in Senegal.

Table 3. Simulation-scenario matrix.

Scenario	Condition	Load	PV	Synchronous generation	Interconnections / reserve	Main objective
S0	Reference / intact network	IEEE 39-bus base case; for Senegal, selected observed operating point	0 MW additional PV; existing PV generation retained in the case studied	Base-case synchronous commitment	Base state of interconnections and reserve	Validate initial AC power flow, voltages, loadings, losses, and balances before any PV sweep
S1	High load / low PV	Upper load level of the dataset or profile studied	Low PV injection; lower step of the solar-availability range	High synchronous commitment expected to meet demand	Case interconnection state; primary reserve explicitly represented	Test thermal and voltage constraints at high load and establish HCstatic
S2	Low load / high PV	Lower load level of the dataset or profile studied	High PV injection, increased to the feasibility boundary	Synchronous commitment reduced as conventional generation is displaced	Imports/exports and reserve recalculated at each point	Test overvoltages, reverse power flows, low local inertia, and HCfrequency
S3	High load / high PV	High load simultaneous with high PV availability	High PV injection, swept until violation	Synchronous commitment consistent with supply-demand balance	Interconnections and reserve according to the operating point	Identify the dominant constraint under combined stress and compare with S1–S2
S4	Cloud ramp / rapid PV loss	Load maintained at the pre-event point	Rapid PV drop; historical PATRP benchmark: 41 MW solar fluctuation; 2025 case: decrease observed on 14 August	Synchronous commitment fixed at the initial instant, followed by simulated primary response	Reserve explicitly represented; sensitivities with/without regional support	Measure RoCoF, nadir, recovery time, and reserve requirement after a rapid PV decrease
S5	Chronological / probabilistic	Load time series	PV time series with forecast errors; 500 samples/point in the benchmark	Variable with time and generation displacement	Time-varying imports/exports and reserve	Quantify Pr(violation), HCprob, and temporal variability of hosting capacity
SEN-AUG2025	Empirical validation, 12–14 August 2025	Actual profile over the period; local Touba–Ndindy peak: 144.14 MW at 23:05 on 13 August (not representative of national load)	Aggregate solar peak: 188 MW at 13:00 on 13 August; PV decrease observed on 14 August under cloud cover	Actual committed fleet to be reconstructed from operator data; no undocumented value is imposed	Spinning reserve maintained at 30 MW during the event; actual interconnection state to be reconstructed	Check model consistency against a stable real event: voltage $\pm 5\%$, frequency 50 ± 0.3 Hz, and no major congestion attributed to renewables

6. Hypotheses and Expected Tests

H1. Static-overestimation hypothesis. Limiting the assessment to voltage and thermal criteria can inflate the secure hosting capacity when added PV curtails synchronous unit commitment. Test: within each scenario, compute HCstatic and, at the identical load level and PV siting, compute HCfrequency. Indicators: Heq, RoCoF, frequency nadir, and FCHCR. The hypothesis is corroborated when HCfrequency is strictly lower than HCstatic; it is not corroborated in cases where AC constraints become binding prior to the frequency constraint.

H2. N–1 penalty hypothesis. Enforcing N–1 security may depress hosting capacity relative to the intact system, contingent on PV placement and operating point. Test: at every PV increment, apply the chosen single-element contingencies and determine HCN–1. Indicators: $\Delta HCN-1$, worst post-contingency voltage, maximum element loading, and the identified critical element. Support is established as soon as a contingency renders a PV increment infeasible that remained admissible in the intact network. If HCN–1 equals HCstatic throughout the tested set, the absence of a penalty is reported without extrapolating to untested

contingencies.

H3. Uncertainty-effect hypothesis. Joint uncertainty in PV output and demand elevates the probability of constraint violations and can lower hosting capacity relative to a deterministic baseline. Test: at each PV level, run Monte Carlo sampling and estimate $\Pr(\text{violation})$ under risk tolerances of 1%, 5%, and 10%. In the benchmark, 500 samples per point with $\sigma_{PV} = 5\%$ and $\sigma_{load} = 1\%$ are used as sensitivity assumptions. Indicators: the $\Pr(\text{violation})$ versus PV curve, HCprob, and the stability interval of the estimator as the sample size grows. The hypothesis is supported whenever HCprob falls below HCstatic at the specified risk level.

H4. Time-dependence hypothesis. Hosting capacity is time-varying as a function of demand, PV availability, synchronous commitment, reserve posture, and interconnection status. Test: compare the boundaries derived for S1, S2, S3, and the chronological series S5 under identical electrical criteria. Indicators: temporal dispersion of HCstatic, HCN-1, HCprob, and HCfrequency, together with the identity of the binding constraint. Evidence supports the hypothesis if either the admissible capacity or the dominant limiting constraint shifts across operating states.

H5. Regional-support hypothesis. OMVS/OMVG interconnections can enhance frequency response by supplying, depending on their operating state, inertial

support and/or deployable primary reserve, while not obviating the requirement for local reserves. Test: rerun the critical frequency contingency while independently varying external inertial contribution, import level, and primary reserve. Indicators: RoCoF, nadir, recovery time, HCfrequency, and the minimum reserve required. PATRP benchmarks, including the historical thermal potential of 63 MW and the studied regional contributions, are employed solely for historical validation and sensitivity analysis and do not prescribe parameters for the 2025 system.

These five hypotheses are evaluated on a scenario-by-scenario basis rather than treated as propositions requiring universal confirmation. Each may be supported, refuted, or remain indeterminate if data are insufficient. For the Senegalese application, no national hosting-capacity value is inferred from the IEEE 39-bus benchmark; national results are presented only when the SPS-2025 model and matching operating data are available and have been verified.

6.1. Historical Benchmarks for Comparison with Simulations

Prior to any interpretation of hosting capacity, the SPS-PATRP model must reproduce documented historical behavior qualitatively and, where data allow, quantitatively. The benchmarks below serve as consistency checks rather than constituting new hosting-capacity findings.

Table 4: Historical PATRP benchmarks used for validation and sensitivity analyses.

Historical quantity	Value / documented case	Use in Article 1
Potential thermal reserve	63 MW	Primary-response sensitivity
Maximum solar fluctuation considered	41 MW	Ramp / reserve test
Maximum wind fluctuation considered	54 MW	Reserve test
Loss of Sendou	115 MW	Severe generation contingency
Maximum wind loss (2022)	72 MW	Renewable-generation contingency
Regional reserve 2022	38 MW (12 OMVS + 26 OMVG)	Validation of interconnection effect
Regional reserve 2028	65 MW (21 OMVS + 44 OMVG)	Regional sensitivity

7. Results-Presentation Framework

7.1. Numerical Protocol and Scope of the Results

This section provides a numerical verification of the proposed methodology on the New England 39-bus test system. Static network data are drawn from MATPOWER case39.^[10] To avoid triviality in the voltage sweep caused by an initial operating point marginally above the upper limit, generator voltage setpoints are capped at 1.045 p.u. for the benchmark. Validation bounds are set to 0.94–1.06 p.u. in the intact configuration and 0.90–1.10 p.u. under emergency conditions. These bounds serve solely as benchmark criteria and are not intended to reflect Senegalese regulatory requirements.

Two PV deployment patterns are examined: (i) a geographically dispersed allocation across buses 4, 8, 15, 20, and 27 in proportion to local demand, and (ii) a fully concentrated injection at bus 4. As PV capacity rises, local synchronous units are progressively backed off. AC power flows are solved at each increment using Newton's method. For N-1 assessment, four representative non-radial line outages are considered, namely 1–2, 1–39, 4–14, and 6–11, with emergency criteria of 0.90–1.10 p.u. for voltages and a 120% thermal loading ceiling. These emergency thresholds are benchmark assumptions and are not to be interpreted as Senelec criteria.

The probabilistic study employs 500 Monte Carlo

samples per operating point, with independent Gaussian relative errors of 5% for PV and 1% for load. These settings constitute class-D sensitivity assumptions. Frequency dynamics are represented by an aggregated primary-response model parameterized with published inertia constants for the IEEE 39-bus system; inertial and regulating support from the external grid is tested at 0%, 25%, 50%, and 100%. The applied severe contingency is a 650 MW loss. Frequency findings should therefore be read as benchmark sensitivity results rather than as a direct prediction of Senegal's frequency nadir. By the same token, the probabilistic outputs are to be interpreted as numerical sensitivity indicators; when reproducing the analysis, increasing the sample size is recommended to enhance robustness, particularly in the vicinity of the

1% and 5% risk thresholds.

7.2. Static AC Hosting Capacity

The base scenario converges with $V_{min} = 0.982$ p.u., $V_{max} = 1.056$ p.u., and a maximum line loading of 78.5%. Under geographically distributed PV, the static hosting capacity reaches 1,400 MW; the next increment triggers the first violation, driven primarily by the upper voltage constraint. When PV is concentrated at bus 4, the binding limit appears earlier, at approximately 1,175 MW, due to thermal loading. In this benchmark, spatial dispersion of PV thus increases the static hosting capacity by about 19% relative to the concentrated placement.

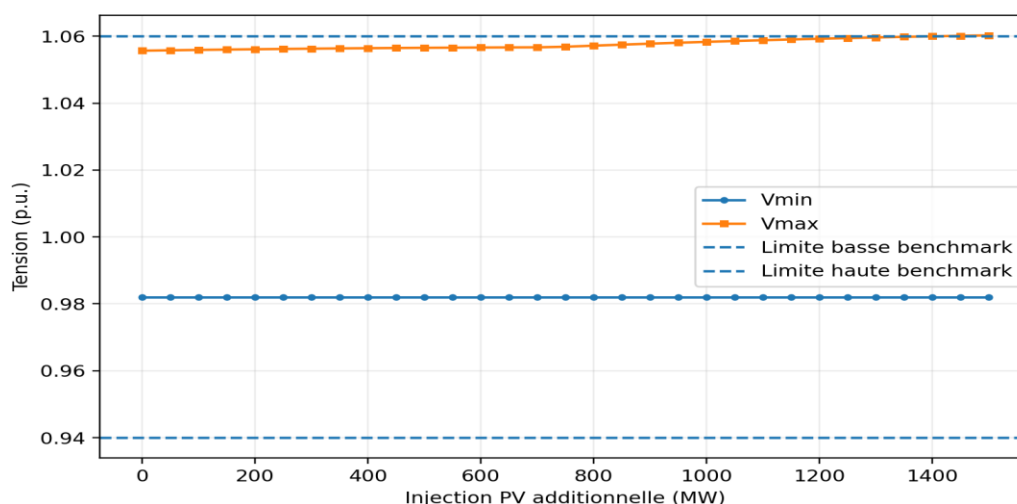


Figure 1: Extreme voltages versus additional PV injection – IEEE 39-bus system, distributed PV.

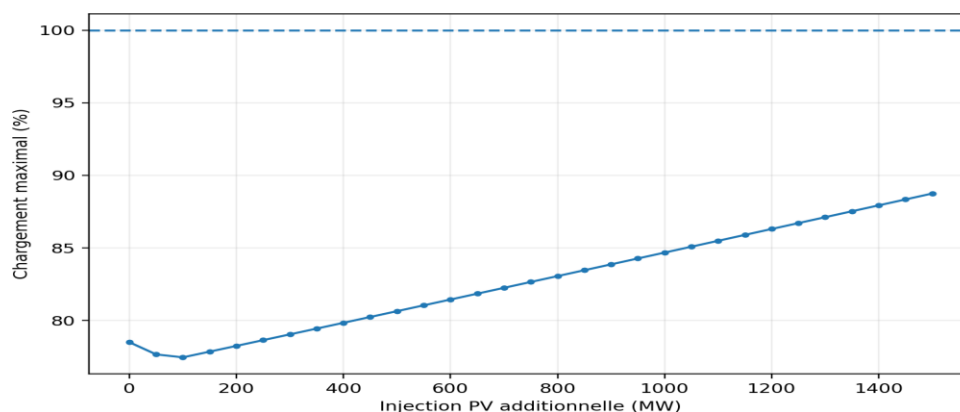


Figure : Maximum branch loading versus additional PV injection – IEEE 39-bus system, distributed PV.

7.3. N-1 Security

Within the examined set of non-radial contingencies, no incremental penalties are triggered prior to the static limit of 1,400 MW under the reference emergency criteria. At 1,400 MW, the maximum post-contingency loading remains below 88% for contingencies 1-2, 1-39, 4-14, and 6-11, and all voltages stay inside the emergency admissibility bounds. Accordingly, $HC_{N-1} = 1,400$ MW for this subset. This outcome does not establish system-wide N-1 security for the IEEE 39-bus network, since several radial generator-connection

branches are excluded; a complete appraisal must explicitly treat islanding and generator-outage events as distinct cases.

7.4. Probabilistic Hosting Capacity

The probability of constraint violations rises sharply in the vicinity of the deterministic limit. With 500 samples per loading level, estimated violation rates are approximately 1.0% at 975 MW, 4.4% at 1,100 MW, 6.0% at 1,125 MW, 9.2% at 1,150 MW, and 13.0% at 1,175 MW. The implied probabilistic hosting

capacities are therefore about 975 MW at a 1% risk level, 1,100 MW at 5%, and 1,150 MW at 10%. At the 5% risk threshold, uncertainty reduces the hosting

capacity by 300 MW, or 21.4%, relative to the 1,400 MW static benchmark.

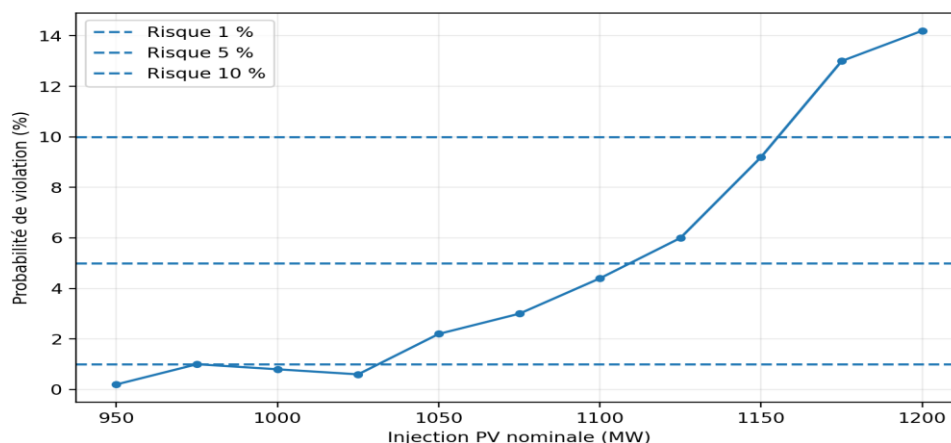


Figure 3. Probability of constraint violation versus PV level – Monte Carlo analysis, IEEE 39-bus system.

7.5. Inertia, RoCoF, and Frequency Nadir

In the absence of external network support, the simulated frequency nadir declines from approximately 49.717 Hz in the base case to 49.698 Hz when about 1,000 MW of PV-equivalent synchronous capacity is displaced, to 49.663 Hz at 1,500 MW, and to 49.529 Hz at 2,000 MW. With 25% equivalent regional support, the nadir at 2,000 MW increases to around 49.706 Hz; with 50% and 100% support, it reaches 49.783 Hz and 49.858 Hz, respectively. These results demonstrate that frequency performance is not governed solely by local PV penetration: the interconnection condition and the volume of synchronous plant actually coupled to the grid exert a decisive influence on both the nadir and the rate

of change of frequency.

The 49.7 Hz level is treated here as an operational benchmark rather than a trigger for under-frequency load shedding. The historical PATRP report identifies 49 Hz as the first UFLS set point. Accordingly, any HCfrequency metric referenced to 49.7 Hz should be regarded as a conservative operating constraint and not conflated with a stability or protection threshold. In the benchmark scenario without regional support, this reference becomes binding once the first synchronous unit is displaced, whereas with at least 25% regional support it remains nonbinding over the static operating range examined.

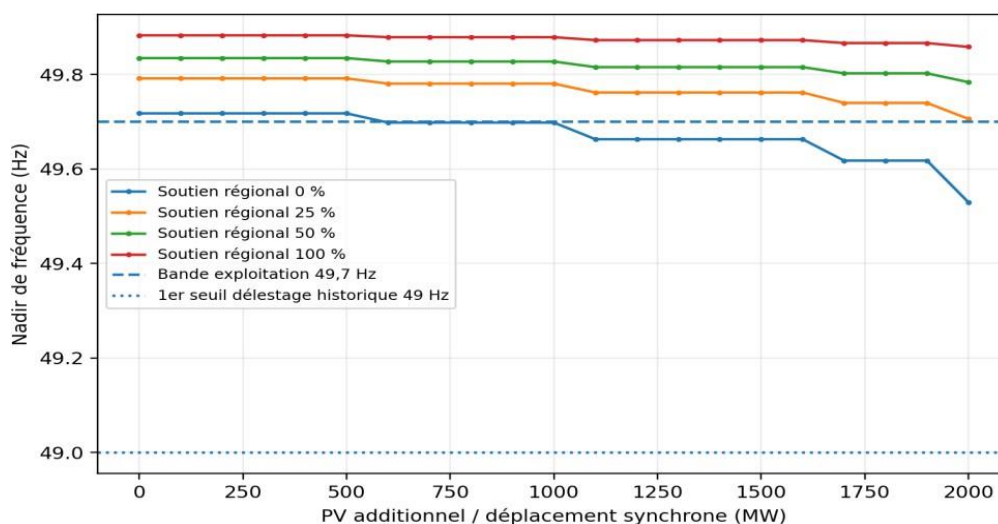


Figure 4. Sensitivity of frequency nadir to displaced synchronous generation and external-network support.

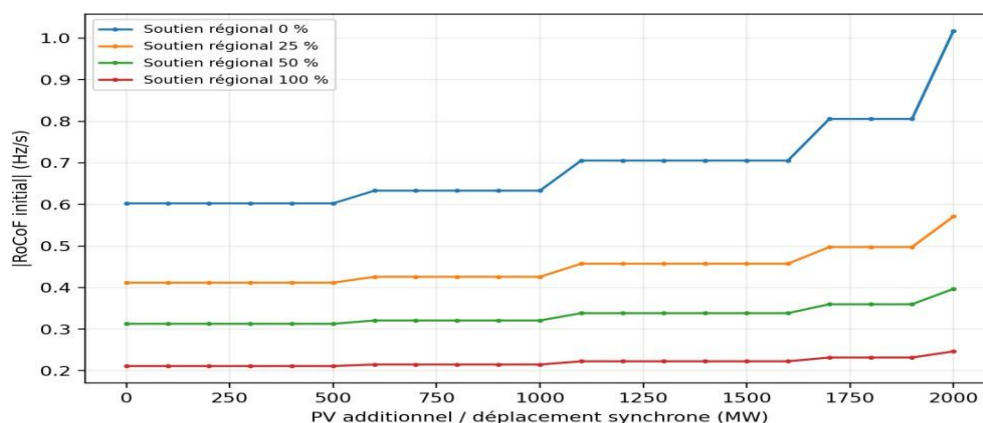


Figure 5: Sensitivity of RoCoF to displaced synchronous generation and external-network support.

7.6. Empirical validation, Senegal, August 2025

Measurements collected from 12 to 14 August 2025 provide an independent empirical cross-check. The daily average system frequencies for the three days were 49.995 Hz, 50.019 Hz, and 50.004 Hz. Corresponding minima were 49.277 Hz, 49.297 Hz, and 49.331 Hz, while the maxima reached 50.565 Hz, 50.602 Hz, and 50.775 Hz. The proportion of time the frequency remained within the 49.7 to 50.3 Hz band was 95.73%, 99.62%, and 96.23%. The operator's report further notes

the absence of significant congestion attributable to renewable generation during this interval and confirms that a spinning reserve of 30 MW was maintained during the Grand Magal. Taken together, these observations substantiate the system's stability over the SEN-AUG2025 window, although they are, by themselves, insufficient to establish a definitive maximum hosting capacity.

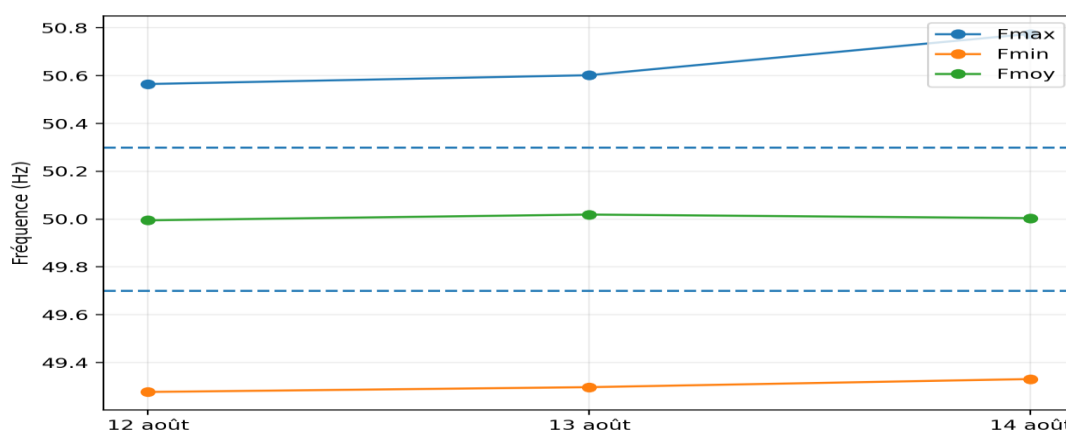


Figure 6: Observed frequency statistics of the Senegalese interconnected power system, 12–14 August 2025 (source: Senelec, Annual Energy Movements Report 2025 [11]).

7.7. Quantitative Summary

The values are not hosting capacities of the Senegalese power system.

Table 5. Summary of numerical results for the IEEE 39-bus benchmark.

Indicator	Benchmark value	Interpretation
Indicator	Benchmark value	Interpretation
HCstatic – distributed PV	1 400 MW	Upper-voltage limit
HCstatic – concentrated PV	≈ 1 175 MW	Thermal limit
HCN-1 – selected contingencies	1 400 MW	No additional penalty before the static limit
HCprob – 1% risk	≈ 975 MW	Assumptions $\sigma_{PV}=5\%$, $\sigma_{load}=1\%$
HCprob – 5% risk	≈ 1 100 MW	21.4% reduction vs HCstatic
HCprob – 10% risk	≈ 1 150 MW	Sensitivity to risk level
Nadir at 2,000 MW, 0% external support	≈ 49,529 Hz	High sensitivity to local inertia
Nadir at 2,000 MW, 25% external support	≈ 49,706 Hz	Stabilizing effect of interconnection

7.8. Main Result and Transferability Limitation

Under the benchmark configuration and a 5% risk criterion, the secure envelope defined by the constraints actually quantified is dictated by uncertainty: $\min(1,400; 1,400; 1,100) = 1,100$ MW, prior to the application of any regulatory frequency requirement. Folding frequency into a single scalar figure is only legitimate once a nadir criterion is fixed. A 49.7 Hz choice yields a conservative operational ceiling that depends acutely on regional support, whereas 49 Hz corresponds to the first documented PATRP load-shedding threshold. It follows that publishing a single HCfrequency number without jointly specifying both the nadir threshold and the interconnection state would not meet scientific standards.

For Senegal, the information currently available is sufficient to validate the operating context, historical reserve levels, load-shedding thresholds, and the SEN-AUG2025 study window; it does not yet amount to a complete and verified SPS-2025 model that includes the requisite impedances, network states, nodal demands, unit commitment and dispatch, and dynamic parameters. Consequently, no HCstatic, HCN-1, HCprob, or HCfrequency figure is posited for Senegal in this paper. All numerical values cited above are results from the IEEE 39-bus benchmark and are used solely to validate the methodological chain before deployment on the Senegalese operator model.

8. DISCUSSION

The benchmark findings support three operational insights. First, siting is consequential: for identical assumptions, geographically dispersing PV across multiple buses raises the static boundary relative to concentrating the same injections at a single bus. Second, uncertainty is not peripheral. At the 5% risk level adopted here, the probabilistic limit becomes the tightest among the quantified constraints. Third, frequency performance is not inferable from connected PV capacity alone; it varies with the synchronous generation displaced by PV and with the level of external support that is actually available. Identical installed PV capacities can therefore correspond to markedly different security margins across operating states.

For Senegal, this cautions against seeking a single PV percentage that would remain valid across all conditions. A more actionable question is which constraint closes the hosting envelope first in a given system state. If HCfrequency lies below HCstatic, a conventional AC study overstates the margin for that scenario; if not, another mechanism binds, be it voltage, thermal loading, contingency performance, or uncertainty. Identifying the binding mechanism is more informative for planning than reporting a single HC value detached from the assumptions that produced it.

Interconnections also require careful interpretation. Historical Senegalese analyses indicate that neighboring systems can provide material inertial support^[13], so

modeling Senegal as an isolated low-inertia system would be misleading. A more faithful representation is a system where local synchronous inertia may fall while the country remains coupled to a regional network. The effective contribution of that network depends on topology, scheduled and unscheduled exchanges, available reserves, and the operating point at the time of disturbance.

The comparison with other West African studies clarifies the incremental contribution. The Niger work shows that full chronological AC assessment is feasible and valuable in a sub-Saharan context.^[12] The present framework extends that analysis at transmission level by integrating N-1 security and frequency response within a unified assessment chain. The intent is to broaden the constraint set considered once inertia and interconnections become salient, not to supplant established HC methodologies.

The PATRP benchmarks also disentangle three effects that are sometimes conflated: instantaneous inertia following a disturbance, primary reserve delivered over the subsequent seconds, and support associated with interconnections.^[13] The benchmark results demonstrate that external support can materially alter the frequency nadir. Senegalese historical data, however, show that such support is conditional on import levels and on the reserves actually available. That distinction should remain explicit when transferring the analysis to the SPS-2025 model.

8.1. Study Limitations and Domain of Validity

The benchmark HC values are conditioned on the specific operating points, dynamic models, contingencies, uncertainty characterizations, and interconnection states examined. PATRP parameters are class-B historical references, not a representation of the 2025 system. Likewise, the voltage, loading, probabilistic-risk, and emergency criteria used are methodological choices or sensitivity assumptions whenever they are not established as regulatory requirements. Each numerical outcome is therefore tied to its scenario, provenance, and assumptions, and no IEEE 39-bus value is carried over to Senegal.

Article 1 is intentionally diagnostic, not prescriptive, with respect to storage. A BESS may appear in validation examples where its operation is documented, but sizing and control optimization are deferred to the subsequent thesis work. This separation preserves an estimate of the intrinsic secure network limit distinct from the corrective action of a flexibility resource.

9. CONCLUSION

This study formulated and validated a hierarchical procedure for PV hosting-capacity assessment that integrates AC steady-state operation, N-1 security, PV and load uncertainty, and frequency response. The IEEE 39-bus benchmark indicates that the resulting boundary

is sensitive to the location of PV connections, the treatment of uncertainty, and the extent of synchronous or external support that persists. Distributed PV attains a static HC of 1,400 MW, compared with about 1,175 MW under a concentrated siting. With a 5% risk level, the probabilistic HC declines to approximately 1,100 MW. Frequency simulations further qualify the boundary: with 2,000 MW of displaced synchronous generation, the nadir shifts substantially as external support is varied.

The Senegalese application warrants caution. The 2025 operating data and PATRP benchmarks confirm the relevance of renewable variability, spinning reserves, regional interconnections, and frequency dynamics to the system.^[11, 13] The 12–14 August 2025 episode provides a useful empirical check, yet it does not justify declaration of a national hosting-capacity value. At this stage, the principal outcome is a validated methodological chain and a clearly specified set of data requirements for full-scale application to a verified SPS-2025 model. Publishing a single national threshold before those inputs are secured would exceed the evidentiary basis of this study.

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